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Stochastic Semidefinite Programming for Emergency Relief Logistics

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Abstract In this work we propose a novel approach to model the logistics networks that prepare for and respond to emergency events in China. Stochastic Semidefinite Programming techniques are deployed to handle the uncertain demands of emergency relief activities and provide insightful recommendations for government and humanitarian organizations on logistics network construction and response activity management.

Keywords emergency relief; logistics; stochastic semidefinite programming

1. Introduction

In January 2008, power failure caused by a heavy snow storm brought much of China to a halt. Hundreds of thousands of people were stranded during the peak traveling season by train delays. Brownouts due to coal shortage affected about half of China's 34 provinces and regions. In May of the same year, one of the worst earthquakes in decades struck central China, killing over 80,000 people. While the damage caused by nature disasters may be inevitable, quicker response and improved relief efforts are crucial to give the population in crisis the aids they need. Unfortunately, the past catastrophes have highlighted the severe difficulties that the government and humanitarian organizations have in planning for, and responding to, emergency events.

Disaster relief logistics is perhaps one of the most challenging supply chain problems as the demand of relief is largely unpredictable. Most of the existing research is conducted under the assumption that a reasonable level of infrastructures for disaster relief has been already put in place. This assumption may be true for many developed countries. But for a developing country like China, such underlying structures and corresponding operation policies are far from being sufficient and satisfactory. The past crises have indicated the urgent need of conducting research to provide insightful recommendations and suggestions on infrastructure construction and logistics operations for disaster relief in the country.

Our work aims to fill this gap in the literature. In this paper, we present a novel approach to model the logistics network required to prepare for and respond to a natural disaster. Stochastic Semidefinite Programming is deployed to deal with the uncertainty of emergency events. Specifically, two critical issues in disaster relief: prestockpiled inventory strategy for emergency response and postdisaster demand management are considered. The logistics network is designed to achieve the optimal performance taking into account the tradeoff between the costs of predisaster preparation and postdisaster response. Our research will integrate existing works on predisaster facility location and postdisaster supply chain management to provide useful insights into the overall design of a framework for disaster relief efforts in the country.

Our research intends to address two fundamental issues: (1) before a disaster strikes, where and how much emergency supplies should be stockpiled; (2) when a disaster strikes and our preparation does not meet the demand, what actions we should take to make up. Note that these two issues are coupled together very tightly. For instance, a better preparation in general leads to a more efficient response. Both preparation and response involve costs and clearly there is a tradeoff between these costs. In addition, since a natural disaster is unpredictable, the demand placed on relief activities is very uncertain. All these challenges make the modeling of the logistics network for disaster preparation and relief very difficult. In this research we adopt Stochastic Semidefinite Programming (SSDP) developed by Ariyawansa and Zhu [4] to tackle this problem. SSDP was introduced as the stochastic counterpart of another very useful optimization model: Semidefinite Programming (SDP) (Boyd et al. [12], Todd [21], Vandenberghe and Boyd [23], Wolkowicz et al. [24]), which has been studied for many years and has many applications in Finance, Engineering, and Statistics. In a traditional SDP model, all the data is assumed to be deterministic. SSDP extends SDP by allowing random data in applications. SSDP can also be considered as a generalization of (two-stage) Stochastic Linear Programming (SLP) in the sense that SSDP allows matrix inequality constraints instead of linear constraints in SLP. Hence, SSDP naturally inherits and extends the applicability of SDP and SLP, and thus becomes a very useful model for applications that involve both data uncertainty and nonlinear constraints.

Let $\mathbb{R}^{m\times n}$ and $\mathbb{R}^{n\vee n}$ denote the vector spaces of real $m\times n$ matrices and real symmetric $n\times n$ matrices respectively. For $U,V\in\mathbb{R}^{n\vee n}$ we write $U\succeq 0$ ($U\succ 0$) to mean that U is positive semidefinite (positive definite), and we use $U\succeq V$ or $V\preceq U$ to mean that $U-V\succeq 0$. As introduced in Ariyawansa and Zhu [4], an SSDP with recourse in dual standard form based on deterministic data $A_i\in\mathbb{R}^{n_1\vee n_1}$ for $i=1,2,\ldots,m_1$, $b\in\mathbb{R}^{m_1}$, and $C\in\mathbb{R}^{n_1\vee n_1}$; and random data $d\in\mathbb{R}^{m_2}$, $W_i\in\mathbb{R}^{n_2\vee n_2}$ for $i=1,2,\ldots,m_1$, $T_i\in\mathbb{R}^{n_2\vee n_2}$ for $i=1,2,\ldots,m_2$, and $D\in\mathbb{R}^{n_2\vee n_2}$ that depend on an underlying outcome ω in an event space Ω with a known probability function P, is defined as:

$$\begin{aligned}\text{maximize}\quad & b^{\mathsf{T}}y+\mathbb{E}[Q(y,\omega)]\\ \text{subject to}\quad & \sum_{i=1}^{m_1} y_iA_i\preceq C,\end{aligned}\tag{1}$$

where $y\in\mathbb{R}^{m_1}$ is the first-stage variable, $Q(y,\omega)$ is the maximum of the problem

$$\begin{aligned}\text{maximize}\quad & d(\omega)^{\mathsf{T}}x\\ \text{subject to}\quad & \sum_{i=1}^{m_1} y_iW_i(\omega)+\sum_{i=1}^{m_2} x_iT_i(\omega)\preceq D(\omega),\end{aligned}\tag{2}$$

where $x\in\mathbb{R}^{m_2}$ is the second-stage variable, and

$$\mathbb{E}[Q(y,\omega)]=\int_\Omega Q(y,\omega)P(d\omega).\tag{3}$$

A collection of applications of SSDP in geometry, sensor networks, RC-circuit design and construction optimization can be found in Ariyawansa and Zhu [27].

The rest of the paper is organized as follows. Section 2 describes the existing work on disaster relief logistics. Section 3 is devoted to our SSDP formulation for disaster relief logistics. Section 4 describes our solution methodology and presents some preliminary numerical results. And the last section contains concluding remarks and some future research directions.

2. Literature Review

Disaster relief logistics has been studied for many years and received a surge interest after the Asian Tsunami in 2004 and Hurricane Katrina in 2005. Most of the studies in disaster relief logistics focus on operational logistical activities in the relief chain with the objective of optimizing the flow of supplies through existing distribution networks.

Knott [15, 16] considered the problem of delivering food items from a distribution center directly to a number of refugee camps via a single mode of transportation. The author developed a linear programming (LP) model to determine the number of trips to each camp to satisfy demand so that the transportation cost is minimized or the amount of food delivered is maximized. Later the author combined operations research heuristics with artificial intelligence techniques to develop a decision support tool for the same problem.

Haghani and Oh [14], Oh and Haghani [19] developed routing and scheduling plans for multiple transportation modes carrying various commodities from multiple points in a disaster relief operation. The authors assumed that the commodity quantities are known. They formulated a multicommodity, multimodal network flow problem with time windows as a large-scale MIP model on a time-space network with the objective of minimizing the sum of the vehicular flow costs, commodity flow costs, supply/demand carry-over costs and transfer costs over all time periods.

Barbarosoglu et al. [6] focused on tactical and operational scheduling of helicopter activities in a disaster relief operation. They decomposed the problem hierarchically into two sub-problems where tactical decisions are made in the top level, and the operational routing and loading decisions are made in the second level. The authors formulated MIP models for the tactical and operational problems, and solved them using an iterative coordination heuristic.

Ozdamar et al. [20] addressed a problem of distributing multiple commodities from a number of supply centers to distribution centers near disaster affected areas. They formulated a multiperiod multicommodity network flow model to determine pick up and delivery schedules for vehicles as well as the quantities of loads delivered on these routes, with the objective of minimizing the amount of unsatisfied demand over time. The structure of the proposed formulation enabled them to regenerate plans based on changing demand, supply quantities, and fleet size. They developed an iterative Lagrange relaxation algorithm and a greedy heuristic to solve the problem. Angelis et al. [2] considered a multidepot, multivehicle routing and scheduling problem for air delivery of emergency supply in Angola. Planes deliver full cargo to individual clients from warehouses in port cities. The authors set a service level for food distribution and developed an integer programming (IP) model that maximizes the total satisfied demand. They provided numerical results for real problem instances.

Beamon and Kotleba [9] developed an inventory management strategy for a warehouse supporting a long-term emergency relief operation. Their analysis was based on a case study of a single humanitarian agency operating a warehouse in Kenya, responding to the complex humanitarian emergency in south Sudan. The authors developed a multisupplier inventory model that optimizes the reorder quantity and reorder level based on the costs of reordering, holding, and back-orders. Beamon and Kotleba [10] continued this work by comparing the performance of three inventory management strategies on the same problem. The authors developed a simulation model and a relief specific performance measurement system to identify system factors that contribute most significantly to overall performance.

Recently there are some studies in the literature that address strategic configuration of the relief chain. For instance, Beamon [7] developed a stochastic inventory control model that determines optimal order quantities and reorder points for a long-term emergency relief response. And they conducted field research with nongovernmental organization World Vision International. Data were collected from its warehouse operations in Lockichoggio, Kenya, as it was responding to the complex humanitarian emergency in south

Sudan. Beamon and Balcik [8] developed a model that determines the number and locations of distribution centers in a relief network and the amount of relief supplies to be stocked at each distribution center to meet the needs of people affected by the disasters. Their model, which is a variant of the maximal covering location model, integrates facility location and inventory decisions, considers multiple item types and captures budgetary constraints and capacity restrictions.

The existing studies fall in two categories. One category focuses on very detailed post disaster network flow operations in a limited area, and the other category considers the logistics problem from a very large scale. There is no work in the literature that addresses the specific issues that the Chinese government faces in disaster prevention and response.

China, as a developing country, is in need of designing and constructing a framework for disaster relief logistics. This framework must be put in place before any specific study focused on network flow, activity management, etc., can be implemented. Moreover, many existing works that focus on global-scaled issues can not be applied directly to China's situation. Our work in this paper strives to seek an appropriate balance between scale and details so that the particular circumstances in China can be dealt with more efficiently.

3. SSDP Formulation

SSDP is an appropriate model for disaster relief logistics. First, SSDP is a stochastic model so it can handle data uncertainty (i.e., relief demand) in our application. Secondly, SSDP allows nonlinear matrix inequality constraints, which is crucial when modeling geographical regions that may be affected by a natural disaster. Finally, the two-staged structure of SSDP fits into the predisaster and postdisaster situation of our application very naturally. In our solutions of the SSDP model for disaster relief logistics, the first stage solution provides suggestions for per-disaster activity and the second stage solution offers advice on postdisaster demand management. And SSDP makes these decisions while taking into account the tradeoff between the predisaster preparation cost and the postdisaster response cost and the uncertainty of the relief activity demand.

With the help of SSDP, we propose to approach the problem in three phases.

• *Phase* I: The regions that might be affected by a future emergency are modeled as ellipses. These ellipses represent the regions that our relief networks are supposed to cover. We use two types of ellipses, fixed and random. Fixed ellipses indicate the areas of strategically importance that must be covered by the relief network. In addition, we use random ellipses to represent the regions that *might* be affected by the emergency event with certain probabilities.

This phase of our work requires extensive collaborations with government and humanitarian organizations on gathering historical data to plot these ellipses. Ellipses are chosen to indicate the disaster affected areas for a number of reasons. First, many disaster affected regions present the shape of an ellipse, e.g., in an earthquake or a fire. Secondly, regions that have no-regular shapes can be easily approximated by a set of ellipses. Finally, covering ellipses can be readily handled by matrix inequalities in the SSDP model.

The online article (George [13]) provides a map of epicenters of historical earthquakes in China. It can be observed that various areas are affected by earthquake with different frequencies. So it would be appropriate to indicate these areas using ellipses with different probabilities in our model.

• *Phase* II: This phase generates our first stage decisions, i.e., predisaster preparation decisions. These decisions include recommendations on the location of the relief facilities and the amount and type of supplies to be stored in the facility. Of course, the trivial solution would be a network that covers all the ellipses, both fixed and random. But in practice, that may be too expensive and even financially impossible. The actual logistics network should cover all the fixed ellipses and almost all the random ellipses with high probabilities and leave the burden of making corrections to the second stage of our model. Mathematically, this phase of our model determines a circle. The center of the circle is the location of the relief center, where relief supplies are prestockpiled. The radius of the circle presents the capacity of the center radiating out to the surrounding area, which is proportional to the amount of supplied prestored in the relief center, and/or the time it takes to deliver supplies to the surrounding area.

• *Phase* III: This phase generates our second stage decisions, i.e., postdisaster response decisions. Due to the uncertain nature of a disaster, it may happen that our preparation is not adequate for the emergency. For instance, a tornado may sweep an area that was not covered by our forecast. Under this situation, our model should be able to provide reasonable remedy advice. Based on the second stage solutions of the SSDP model, we can then derive high level guidance on remedy actions under the situation that our previous recommendations fail to meet the demand of the actual emergency. All possible scenarios of the crisis are considered and the corresponding remedy is created for each realization of the random event. Mathematically, in this stage, the radius of the initial circle defined in the first stage is allowed to be enlarged (meaning that our model allows the capacity of the relief center to be enhanced by certain means, which involves extra costs though) in order to cover the demand that was not satisfied by the original plan.

Note that our model determines the location and the inventory of *one* relief center that supports its surrounding area. In the case that multiple centers are required, the model needs to be performed for each center separately.

The first stage decision and second stage decision are tightly connected together in the SSDP model. Decisions in both stages involve costs and there is a trade off between them. Additionally, these decision are made under uncertainty. The SSDP model make these decisions so that the expected total cost is minimized and the trade off between the first stage cost and second stage cost is optimized.

The parameters in the model can be adjusted to reflect our concerns on various factors. For example, a particular realization of the random ellipse may represent a rural area that is very difficult to reach and has very sparse population distribution. Then we can assign a relatively high weight to the corresponding cost parameter to emphasize the fact that it would be very expensive for our network to cover that region. Time constraints are also considered in the second stage decision making. In fact, the second stage decisions are invoked only if the first stage actions fail to meet the demand of the emergency event.

We now present the mathematical details of our model. Suppose that we are given n_f fixed ellipses $\mathcal{E}_i := \{x \in \mathbb{R}^n\colon x^\mathsf{T} H_i x + 2g_i^\mathsf{T} x + v_i \leq 0\} \subset \mathbb{R}^n$, $i = 1, 2, \ldots, n_f$. Here $H_i \in \mathbb{R}^{n \vee n}$, $H_i \succ 0$, $g_i \in \mathbb{R}^n$, and $v_i \in \mathbb{R}$ for $i = 1, 2, \ldots, n_f$ are deterministic data. We are also given n_r *random* ellipses $\tilde{\mathcal{E}}_i(\omega) := \{x \in \mathbb{R}^n\colon x^\mathsf{T} \widetilde{H}_i(\omega) x + 2\tilde{g}_i(\omega)^\mathsf{T} x + \tilde{v}_i(\omega) \leq 0\}$, $i = 1, 2, \ldots, n_r$. Here for $i = 1, 2, \ldots, n_r$, $\widetilde{H}_i(\omega) \in \mathbb{R}^{n \vee n}$, $\widetilde{H}_i(\omega) \succ 0$, $\tilde{g}_i(\omega) \in \mathbb{R}^n$, $\tilde{v}_i(\omega) \in \mathbb{R}$ are random data that depend on an underlying outcome ω in an event space Ω with a known probability function P.

Suppose that at present we do not know the realizations of the n_r random ellipses, and suppose that at some point in time in the future the realizations of these n_r ellipses become known. Also suppose that we need to determine a circle that contains all n_f fixed ellipses and the realizations of the n_r random ellipses. However, this decision needs to be made *before* the realizations of the random ellipses become available. Therefore, when the realizations of the random ellipses do become available, the circle that has already been determined may or may not contain all the realized random ellipses. We assume that at that stage we are allowed to change the radius of the circle (but not its center), if necessary, in order to insure that the modified circle contains all (fixed and realizations of random) ellipses.

We assume that the cost of choosing the circle has three components: the cost of the center, which is proportional to the Euclidean distance to the center from the origin (the initial location of the relief supplies); the cost of the initial radius, which is proportional to the square of the radius (it is in turn proportional to the area of the covered region);

and the cost of changing the radius after the realizations of the random ellipses become available, which is proportional to the increase in the square of the radius. The location and the inventory of the relief center are determined so that the expected total cost is minimized.

Our first goal is to determine $\bar{x} \in \mathbb{R}^n$ and $\gamma \in \mathbb{R}$ such that the circle $\mathcal{B}$ defined by

$$\mathcal{B} := \left\{ x \in \mathbb{R}^n \colon x^\mathsf{T} x - 2\bar{x}^\mathsf{T} x + \gamma \leq 0 \right\}$$

contains the fixed ellipses $\mathcal{E}_i$ for $i = 1, 2, \ldots, n_f$, or equivalently (Boyd et al. [12], Yıldırım [25]) if and only if there is $\tau_i \geq 0$ such that

$$\begin{bmatrix} I & -\bar{x} \\ -\bar{x}^\mathsf{T} & \gamma \end{bmatrix} \preceq \tau_i \begin{bmatrix} H_i & g_i \\ g_i^\mathsf{T} & v_i \end{bmatrix}, \quad i = 1, 2, \ldots, n_f.$$

As indicated above this determination is made before the realizations of the random ellipses $\tilde{\mathcal{E}}_i(\omega)$ for $i = 1, 2, \ldots, n_r$ become known. Note that the center of $\mathcal{B}$ is $\bar{x}$ and that the square of the radius of $\mathcal{B}$ is $\bar{x}^\mathsf{T}\bar{x} - \gamma$. We introduce the following constraints

$$\begin{bmatrix} d_1 I & \bar{x} \\ \bar{x}^\mathsf{T} & d_1 \end{bmatrix} \succeq 0 \tag{4}$$

and

$$\begin{bmatrix} I & \bar{x} \\ \bar{x}^\mathsf{T} & d_2 + \gamma \end{bmatrix} \succeq 0. \tag{5}$$

By Schur Complements (4) is equivalent to $d_1 - \bar{x}^\mathsf{T}(d_1 I)^{-1}\bar{x} \geq 0$, which is in turn equivalent to $d_1 \geq \sqrt{\bar{x}^\mathsf{T}\bar{x}}$. Similarly, constraint (5) is equivalent to $d_2 + \gamma - \bar{x}^\mathsf{T} I^{-1} \bar{x} \geq 0$ and in turn to the constraint $d_2 \geq \bar{x}^\mathsf{T}\bar{x} - \gamma$. Thus, d_1 is an upper bound on the distance between the center of the circle $\mathcal{B}$ and the origin, say $\sqrt{\bar{x}^\mathsf{T}\bar{x}}$, and d_2 is an upper bound on square of the radius of the circle $\mathcal{B}$, say $\bar{x}^\mathsf{T}\bar{x} - \gamma$. Here we use d_1 and d_2 to convert nonlinear terms that will appear in the objective function into SDP constraints.

When the realizations of the random ellipses become available, if necessary, we determine $\widetilde{\gamma}$ so that the new circle

$$\widetilde{\mathcal{B}} := \{ x \in \mathbb{R}^n \colon x^\mathsf{T} x - 2\bar{x}^\mathsf{T} x + \widetilde{\gamma} \leq 0 \}$$

contains all the realizations of the random ellipses. This new circle $\widetilde{\mathcal{B}}$ has the same center $\bar{x}$ as $\mathcal{B}$ but a larger radius, $\tilde{r} = \sqrt{\bar{x}^\mathsf{T}\bar{x} - \widetilde{\gamma}}$. We note that $\tilde{r}^2 - r^2 = (\bar{x}^\mathsf{T}\bar{x} - \widetilde{\gamma}) - (\bar{x}^\mathsf{T}\bar{x} - \gamma) = \gamma - \widetilde{\gamma}$, and thus we introduce the constraint

$$0 \leq \gamma - \widetilde{\gamma} \leq z,$$

where z is an upper bound of $\tilde{r}^2 - r^2$. Let $\bar{c} > 0$ denote the cost per unit of the Euclidean distance between the center of the circle $\mathcal{B}$ and the origin, $\alpha > 0$ be the cost per unit of the square of the radius of $\mathcal{B}$, and let $\beta > 0$ be the cost per unit increase of the square of the radius if it becomes necessary after the realizations of the random ellipses are available.

We define the following decision variables

$$x := [d_1, d_2, \bar{x}^\mathsf{T}, \gamma, \tau^\mathsf{T}]^\mathsf{T} \qquad \text{and} \qquad y := [z, \widetilde{\gamma}^\mathsf{T}, \delta^\mathsf{T}]^\mathsf{T},$$

where except for $\tau \in \mathbb{R}^{n_f}$ and $\delta \in \mathbb{R}^{n_r}$ the other components are as specified above. We also introduce the following unit cost vectors

$$c := [\bar{c}, \alpha, 0, 0, 0]^\mathsf{T} \qquad \text{and} \qquad q := [\beta, 0, 0]^\mathsf{T}.$$

Then we get the model

$$\begin{aligned} \text{minimize} \quad & c^\top x + \mathbb{E}[Q(x,\omega)] \\ \text{subject to} \quad & \begin{bmatrix} I & -\bar{x} \\ -\bar{x}^\top & \gamma \end{bmatrix} \preceq \tau_i \begin{bmatrix} H_i & g_i \\ g_i^\top & v_i \end{bmatrix}, \quad i = 1,2,\ldots,n_f, \\ & 0 \le \tau \\ & 0 \preceq \begin{bmatrix} d_1 I & \bar{x} \\ \bar{x}^\top & d_1 \end{bmatrix}, \\ & 0 \preceq \begin{bmatrix} I & \bar{x} \\ \bar{x}^\top & d_2 + \gamma \end{bmatrix}, \end{aligned} \tag{6}$$

where $Q(x,\omega)$ is the minimum of the problem

$$\begin{aligned} \text{minimize} \quad & q^\top y \\ \text{subject to} \quad & \begin{bmatrix} I & -\bar{x} \\ -\bar{x}^\top & \widetilde{\gamma} \end{bmatrix} \preceq \delta_i \begin{bmatrix} \widetilde{H}_i(\omega) & \tilde{g}_i(\omega) \\ \tilde{g}_i^\top(\omega) & \tilde{v}_i(\omega) \end{bmatrix}, \quad i = 1,2,\ldots,n_r, \\ & 0 \le \delta, \\ & \gamma - \widetilde{\gamma} \le z, \\ & \widetilde{\gamma} - \gamma \le 0, \end{aligned} \tag{7}$$

and

$$\mathbb{E}[Q(x,\omega)] = \int_\Omega Q(x,\omega) P(d\omega). \tag{8}$$

Problem (6, 7, 8) is a two-stage model and the randomness in the data is handled in the second stage, which is impossible with a deterministic approach.

4. Solution Methodology

We propose two methods of solving the SSDP model.

Direct Method: Under the assumption that all random variables have discrete distributions, an SSDP can be converted to an equivalent large-scale Deterministic Semidefinite Program (DSDP). And there have been a number of software packages developed for solving DSDPs (Benson et al. [11], Toh et al. [22]). So in this method, we solve our SSDP model as a DSDP under the assumption that all the random variables have discrete distributions, using the software package SDPT3 developed by Toh et al. [22].

Let $\{(d^{(k)}, (W_i^{(k)}\colon i = 1,2,\ldots,m_1), (T_i^{(k)}\colon i = 1,2,\ldots,m_2), D^{(k)})\colon k = 1,2,\ldots,K\}$ be the possible realizations of the random data and let

$$\begin{aligned} p_k &:= P\big((d(w), (W_i(w)\colon i = 1,2,\ldots,m_1), (T_i(w)\colon i = 1,2,\ldots,m_2), D(w))\big) \\ &= \big(d^{(k)}, (W_i^{(k)}\colon i = 1,2,\ldots,m_1), (T_i^{(k)}\colon i = 1,2,\ldots,m_2), D^{(k)}\big) \end{aligned}$$

be the associated probability for $k = 1,2,\ldots,K$.

Then Problem (1, 2, 3) is equivalent to

$$
\begin{aligned}
\text{maximize}\quad & b^{\mathsf{T}}y + p_1(d^{(1)})^{\mathsf{T}}x^{(1)} + p_2(d^{(2)})^{\mathsf{T}}x^{(2)} + \cdots + p_K(d^{(K)})^{\mathsf{T}}x^{(K)} \\
\text{subject to}\quad & \sum_{i=1}^{m_1} y_i A_i \preceq C, \\
& \sum_{i=1}^{m_1} y_i W_i^{(1)} + \sum_{i=1}^{m_2} x_i^{(1)} T_i^{(1)} \preceq D^{(1)}, \\
& \vdots \qquad\qquad \ddots \qquad\qquad \preceq \quad \vdots \\
& \sum_{i=1}^{m_1} y_i W_i^{K} + \sum_{i=1}^{m_2} x_i^{(K)} T_i^{(K)} \preceq D^{(K)},
\end{aligned}
\tag{9}
$$

where $y \in \mathbb{R}^{m_1}$ and $x \in \mathbb{R}^{m_2}$ for $k = 1, 2, \ldots, K$ are the variables.

Problem (9) can be written as an DSDP in the dual standard form. First let

$$
\begin{aligned}
\bar{b} &:= \left[b^{\mathsf{T}}, p_1(d^{(1)})^{\mathsf{T}}, p_2(d^{(2)})^{\mathsf{T}}, \ldots, p_K(d^{(K)})^{\mathsf{T}}\right]^{\mathsf{T}} \in \mathbb{R}^{m_1+Km_2}; \\
\bar{D} &:= \operatorname{diag}\left(C, D^{(1)}, D^{(2)}, \ldots, D^{(K)}\right) \in \mathbb{R}^{(n_1+Kn_2)\vee((n_1+Kn_2)}; \\
\bar{y} &:= \left[y^{\mathsf{T}}, (x^{(1)})^{\mathsf{T}}, (x^{(2)})^{\mathsf{T}}, \ldots, (x^{(K)})^{\mathsf{T}}\right]^{\mathsf{T}} \in \mathbb{R}^{m_1+Km_2}.
\end{aligned}
$$

Next for $i = 1, 2, \ldots, m_1 + Km_2$, we define $\bar{A}_i \in \mathbb{R}^{((n_f+3(n+1))K)\vee(n_f+3(n+1)+(n_f+n+3)K)}$ as follows:
for $i = 1, 2, \ldots, m_1$,

$$
\bar{A}_i = \begin{bmatrix} A_i & \bar{0} & \cdots & \bar{0} \\ W_i^{(1)} & \bar{0} & \cdots & \bar{0} \\ \vdots & \vdots & \ddots & \vdots \\ W_i^{(K)} & \bar{0} & \cdots & \bar{0} \end{bmatrix};
$$

for $i = m_1 + 1, m_1 + 2, \ldots, m_1 + m_2$,

$$
\bar{A}_i := \begin{bmatrix} \bar{0} & \bar{0} & \cdots & \bar{0} \\ \bar{0} & T_i^{(1)} & \cdots & \bar{0} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{0} & \bar{0} & \cdots & \bar{0} \end{bmatrix};
$$

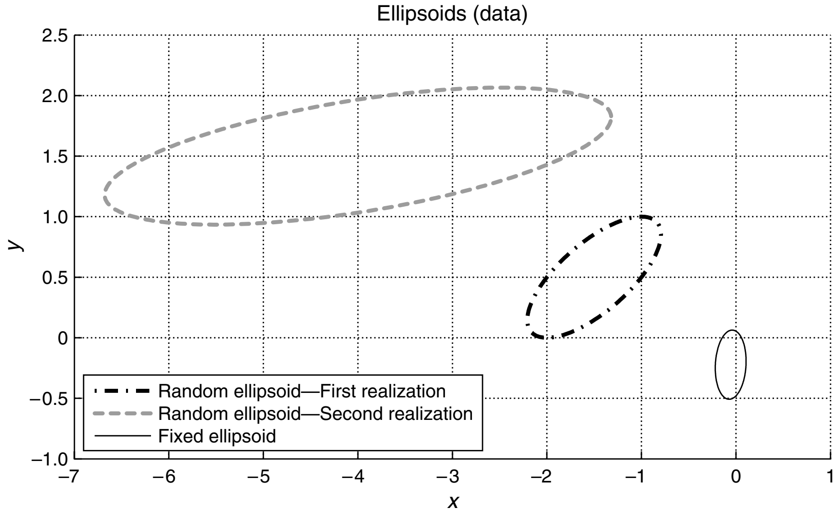
and for $i = m_1 + m_2 \cdot j + 1, m_1 + m_2 \cdot j + 2, \ldots, m_1 + m_2 \cdot (j+1)$, where $j = 1, 2, \ldots, K$,

$$
\bar{A}_i := \begin{bmatrix} \bar{0} & \bar{0} & \cdots & \bar{0} \\ \bar{0} & \bar{0} & \cdots & \bar{0} \\ \vdots & \vdots & T_i^{(j)} & \vdots \\ \bar{0} & \bar{0} & \cdots & \bar{0} \end{bmatrix}.
$$

With these assignments, the problem (9) becomes

$$
\begin{aligned}
\text{maximize}\quad & \bar{b}^{\mathsf{T}}\bar{y} \\
\text{subject to}\quad & \sum_{i=1}^{m_1+Km_2} \bar{y}_i \bar{A}_i \preceq \bar{D}.
\end{aligned}
\tag{10}
$$

FIGURE 1. Sample data set with one fixed ellipse and two random ellipses.



Once our model is in the form of (10), it can solved using the packages available for solving DSDPs (Benson et al. [11], Toh et al. [22]).

Decomposition-Based Interior Point Methods: In Mehrodra and Ozevin [17], Mehrodra and Ozevin introduced log-barrier decomposition algorithms for SSDP. Ariyawansa and Zhu extended their work to propose a new class of volumetric barrier decomposition algorithms (Alizadeh [1], Anstreicher [3], Nesteriv and Nemirovski [18], Zhao [26]) for SSDP and proved polynomial complexity of short-step and long-step members of the class in Ariyawansa and Zhu [5]. The implementation of the mentioned algorithms and the development of corresponding software packages for solving SSDP are still under going.

5. Preliminary Numerical Analysis

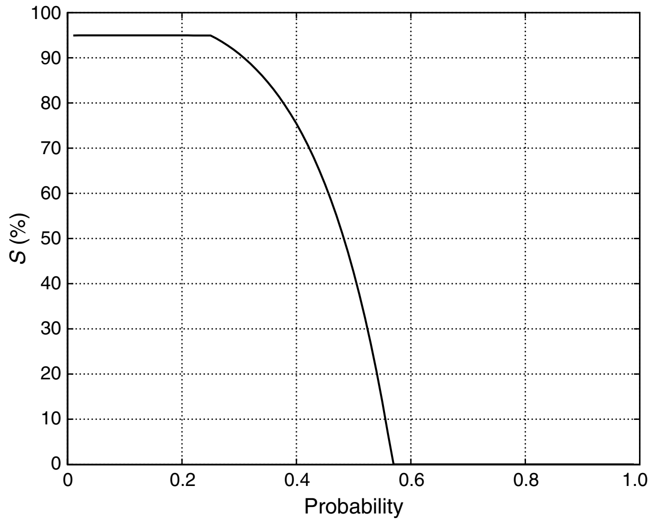
The idea of this analysis is to observe how our SSDP model responds to variations of the input parameter, e.g., preparation (first stage) costs, responses (second stage) costs, and probabilities of the random regions. Due to the limitation of document length, we can only present the simulation results on varying the probability of a particular random region.

The simulations are conducted on the sample data set as shown in Figure 1. We define a performance metric termed percentage saving. It is the saving offered by our SSDP model compared to the worst case solution in which the logistics network needs to cover all fixed and random ellipses. We vary the probabilities of one random ellipse and observe the response of our algorithm. The probability of the occurrence of the further ellipsoid (second realization) is varied from 0 to 1 in increments of 0.01. All the cost parameters are set to equal during this analysis. In Figure 2 we can see that as the probability of the occurrence of the further ellipsoid increases, the percentage saving decreases. This is quite rational, since as the further ellipsoid becomes more probable, the first stage solution likes to cover it in order to minimize the total cost. So the first stage solution approaches the worst case solution as the probability of the further ellipsoid goes to 1, and hence the percentage savings decreases and hit 0 eventually.

6. Conclusion and Future Work

In this study, our objective was to develop a novel approach to model the logistics networks required to prepare for and respond to a natural disaster in China. Stochastic Semidefinite Programming was employed to deal with the uncertainty of emergency events. Specifically,

FIGURE 2. Percentage of savings while varying the probability of an ellipse.



two critical issues in disaster relief: prestockpiled inventory strategy for emergency response and postdisaster demand management were considered. The logistics network was designed to the optimal while taking into account the tradeoff between the costs of predisaster preparation and postdisaster response. Preliminary numerical studies were performed to show that SSDP was a viable and useful model for disaster relief logistics in China.

Our research has provided a framework for disaster-logistics planning. Within this framework, many strategies that focus on specific aspects of the problem can be incorporated to achieve better performance and efficiency. For example, facility localization technique can be imbedded into our first stage decision, and max flow strategies can be incorporated in the second stage decision and so on. It will be interesting to explore these possibilities.

The model we considered in this project has two stages, which provides guidance on pre- and postdisaster activities. It is appropriate for the case that a region is affected by a disaster for only once. In many situations, a same type of disaster may affect a region for more than one time, e.g., in certain season, tornados may sweep an area for multiple times within a period of time. A multistage model might be more appropriate for this new setting.

We will conduct a comprehensive numerical analysis to assess our approach using collected data from China. Simulations will also be used to test the robustness and reliability of our model. We will compare the results of our model with historical records to evaluate the performance of our logistics strategies.

The nature of this research demands and stimulates engagement and collaboration among researchers, government agencies, relief organizations and private corporations. Although we motivate our research from a China's perspective, we believe our result can be used elsewhere in many other countries. The work will have a significant positive impact to our community. The results of our work will certainly help governments in emergency response policy making. Our analysis and modeling will be helpful to government authorities and humanitarian organizations in infrastructure and logistics operations for emergency relief. And in a long run, we propose to develop a user-friendly software interface on emergency relief logistics, and offer training workshops to governmental agencies and relief organizations to use the software.

The humanitarian logistics operations are very different from business logistics and supply chains. The analysis, design, and methodology are more generic and flexible. So it is possible to adapt our model to reflect market demands for business products, then the methods we introduce in this research will become useful logistics tools for business and industries. For example, many chain stores run distribution centers to support demands from retail stores. And our research can provide cost optimal suggestions on the locations and inventory of the distribution centers.

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